MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

ETM7166 – DIGITAL SIGNAL PROCESSING SYSTEMS AND DESIGN IN TELECOMMUNICATIONS
(All sections / Groups)

24 OCTOBER 2019
1:00 PM – 4:00 PM
(3 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 12 pages with 4 Questions only.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please print all your answers in the Answer Booklet provided.

Question 1

(a)   Digital signal processing (DSP) has been used in a wide range of applications.

   (i)   Briefly describe two advantages of DSP compared to analogue signal processing.   [4 marks]

   (ii)   Briefly describe the options available to implement DSP systems in practice.   [4 marks]

   (iii)   Figure Q1.1 shows the spectrum of a bandlimited analogue signal. Illustrate the spectrum of a sampled version of this signal if ideal sampling is performed at a rate higher than the Nyquist rate.
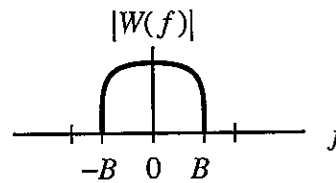   [4 marks]
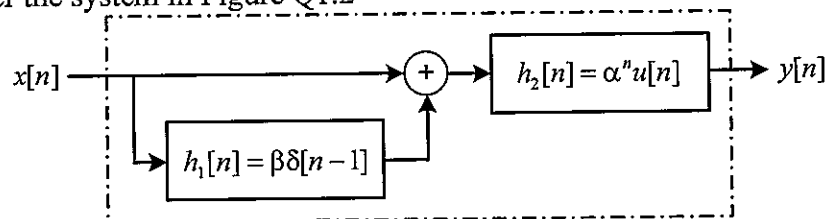

Figure Q1.1

(b)   Consider the system in Figure Q1.2

Figure Q1.2

   (i)   Find the impulse response $h[n]$ of the overall system.   [4 marks]

   (ii)   Is the system causal? Justify your answer.   [2 marks]

(c)   Consider the following system function:

$$H(z) = \frac{5 - 1.8z^{-1}}{1 - 0.8z^{-1} + 0.12z^{-2}} = \frac{2}{(1 - 0.2z^{-1})} + \frac{3}{(1 - 0.6z^{-1})}$$

   (i)   Determine the linear constant coefficient difference equation (LCCDE).
                                                                                  [2 marks]

   (ii)   Determine the impulse response of the system $h[n]$, assuming the system is causal.   [3 marks]

   (iii)   Determine whether the filter is realizable.   [2 marks]

Continued...

Question 2

(a) (i) Briefly discuss the differences between the discrete-time Fourier transform (DTFT) and the discrete Fourier transform (DFT).

[4 marks]

(ii) Figure Q2 shows a practical approach to performing spectral analysis on a continuous-time signal $g(t)$ by computing the DTFT, $G(e^{j\omega})$ of its discrete-time equivalent $g[n]$. The DTFT is usually approximated by the DFT operation. In this context, explain the difference between *high-density spectrum* and *high-resolution spectrum*.   [4 marks]
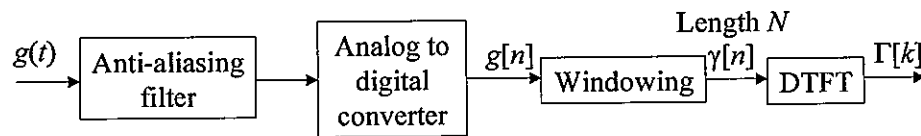

Figure Q2

(b) In the context of designing digital finite impulse response (FIR) filters, answer the following questions.

(i) Explain the rationale of truncating the impulse response of an ideal FIR digital filter.   [2 marks]

(ii) With the aid of a diagram, explain the effects caused by this truncation. Suggest a method to minimize them.   [3 marks]

(c) A signal is sampled at a frequency of 1 kHz, has a useful content from 0 to 100 Hz and is corrupted with noise from 400 to 500 Hz. Design a digital FIR low-pass filter based on a fixed window to attenuate the noise by at least 53 dB without affecting the useful content by more than 3 dB.   [7 marks]

(d) A low pass digital filter has the following characteristics (equiripple passband, but monotone stopband):

$$0.95 \le |H(e^{j\omega})| \le 1, \qquad 0 \le \omega \le 0.3\pi$$

$$|H(e^{j\omega})| \le 0.15, \qquad 0.4\pi \le \omega \le \pi$$

By using the bilinear transformation method, determine the minimum order for the filter.   [5 marks]

Continued...

Question 3

(a)   Explain 3 reasons why we need to consider many different structures when designing discrete-time systems.

[6 marks]

(b)   Figure Q3(a) shows a non-canonic implementation of an Infinite Impulse Response (IIR) filter. Suggest a simple modification in the structure in order to make the system canonic.
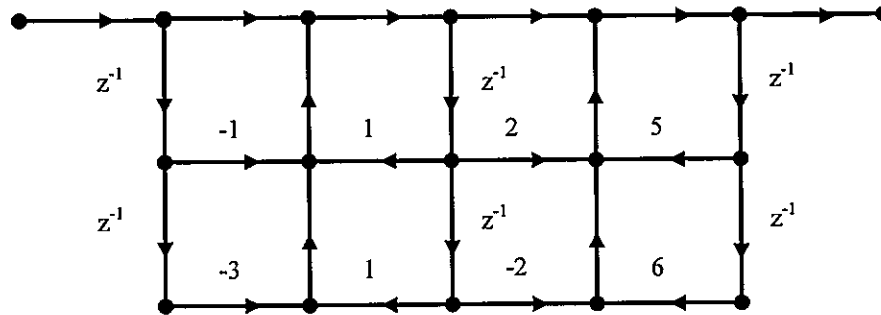

Fig. Q3(a)

[3 marks]

(c)   Consider the interpolator structure in Fig. Q3(b). The filter $H(z)$ has a transfer function $H(z) = 3z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} + 3z^{-5} + z^{-6} + 5z^{-7} + 5z^{-8}$
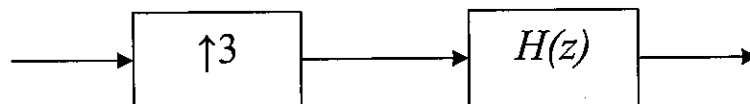

Fig. Q3(b)

(i)   The multirate system will be more efficient if the filter $H(z)$ is represented in polyphase form. Derive the 3-branch polyphase decomposition of $H(z)$ for the above system. Show explicitly the transfer function of each subfilter $E_m(z)$.   [3 marks]

(ii) Realize the system in Fig. Q3(b) using a 3-branch polyphase decomposition. Design your system so that the computational complexity will be lowest.   [2 marks]

(d)   Digital audio tape (DAT) drives use a sampling frequency of 48kHz, whereas compact disc (CD) players operate at a rate of 44.1kHz. In order to record directly from a CD onto a DAT, it is necessary to convert the sampling rate from 44.1kHz to 48kHz.

(i)   Draw the block diagram of an efficient sampling rate converter for the system.   [2 marks]

Continued...

(ii) Determine the frequency response of the filter necessary to suppress imaging and ensure the absence of aliasing in the system.

[2 marks]

(iii) Suggest whether FIR or IIR filter is more computationally efficient for the above sampling rate conversion. Explain your answer.

[3 marks]

(e) A noisy speech signal, after passing through an unknown system, manages to have most of its noises removed. With the aid of a suitable diagram, suggest how adaptive filters can be used to identify the properties of the unknown system.

[4 marks]

Question 4

(a) Figure Q4(a) shows the block diagram of a Linear Prediction Coding (LPC) decoder. Explain how the bit stream is decoded into a synthetic speech.
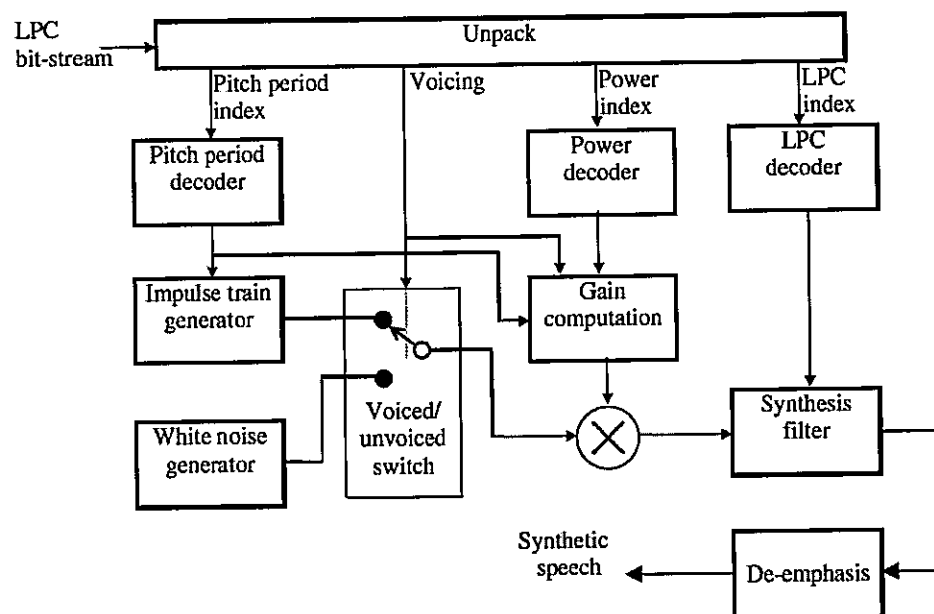

Figure Q4(a)

[8 marks]

(b) Briefly describe the use of energy, zero crossing rates, and prediction gain in the voicing detector of an LPC model.   [6 marks]

(c) Figure Q4(b) shows the plot of a voiced segment with its corresponding autocorrelation function. Explain how the pitch period can be estimated from each plot. Estimate the pitch period.   [3 marks]
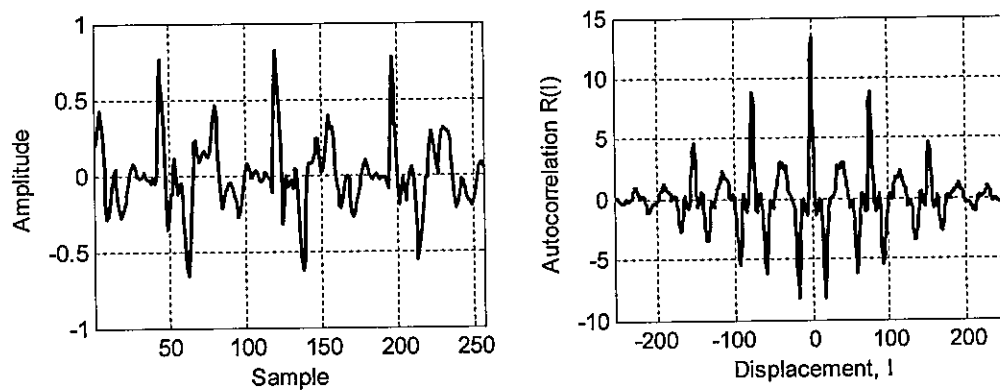
Continued...

Figure Q4(b)

(d) A 256-sample voiced segment, with 16 bits/sample, is to be coded using parametric coding. The linear prediction filter employed is of 14th order, with each coefficient represented by 4 bits, and a combined pitch period, power and voicing bit of 8 bits. Determine the compression ratio achieved by this parametric coder. [3 marks]

(e) In linear prediction applications, it is required to estimate the current sample of the input sequence as a linear combination of the past M input samples. With the aid of a diagram, explain how adaptive filters can be used to achieve this. [5 marks]

End of Questions

Appendix: Formula Sheet

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n] + by[n]$	$aX(z) + bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n] * y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n - n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\dfrac{1}{1 - z^{-1}}$	$\lvert z \rvert > 1$
$-u[-n-1]$	$\dfrac{1}{1 - z^{-1}}$	$\lvert z \rvert < 1$
$a^n u[n]$	$\dfrac{1}{1 - az^{-1}}$	$\lvert z \rvert > \lvert a \rvert$
$-a^n u[-n-1]$	$\dfrac{1}{1 - az^{-1}}$	$\lvert z \rvert < \lvert a \rvert$
$na^n u[n]$	$\dfrac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z \rvert > \lvert a \rvert$
$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z \rvert < \lvert a \rvert$

Closed-form Expression for Some useful Series

$$\sum_{n=0}^{N-1} a^n = \frac{1 - a^N}{1 - a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad \lvert a \rvert < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad \lvert a \rvert < 1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

Continued...

FIR Filter Design

Ideal bandpass

$$h[n] = \frac{w_2}{\pi} \operatorname{sinc}\left(\frac{w_2(n - M/2)}{\pi}\right) - \frac{w_1}{\pi} \operatorname{sinc}\left(\frac{w_1(n - M/2)}{\pi}\right),$$

$$n = 0, 1, \ldots, M$$

Fixed windows

Window	Window function
Rectangular	$w[n] = 1$
Hann	$w[n] = 0.5 - 0.5\cos(2\pi n/M)$
Hamming	$w[n] = 0.54 - 0.46\cos(2\pi n/M)$
Blackman	$w[n] = 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M)$

Window	Passband ripple $20\log_{10}\delta_p$	Stopband attenuation $20\log_{10}\delta_s$	Transition width $|w_p - w_s|$
Rectangular	-13	-21	$1.8\pi/M$
Hann	-31	-44	$6.2\pi/M$
Hamming	-41	-53	$6.6\pi/M$
Blackman	-57	-74	$11\pi/M$

Kaiser window

$$w[n] = \frac{I_0\big(\beta(1 - (n/\alpha - 1)^2)^{0.5}\big)}{I_0(\beta)}$$

$$\beta = \begin{cases} 0.1102(A - 8.7), & A > 50 \\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21), & 21 \le A \le 50 \\ 0, & A < 21 \end{cases}$$

$$M = \begin{cases} (A - 7.95)/(2.285\Delta w), & A \ge 21 \\ 5.655/\Delta w, & A < 21 \end{cases}$$

Optimal Filter Design (Parks-McClellan Algorithm) Estimated Filter Order

$$N = \frac{-20\log\sqrt{\delta_p\delta_s} - 13}{14.6\Delta f}$$

Continued...

CLASSIFICATION OF LINEAR-PHASE FIR SYSTEMS

	$h[n]$ symmetric: $h[n] = h[N-n]$	$h[n]$ antisymmetric: $h[n] = -h[N-n]$
N even	**Type I Linear Phase Filter** $H(e^{j\omega}) = e^{-jN\omega/2} \sum_{k=0}^{N/2} a[k]\cos(k\omega)$ $a[0] = h[N/2]$ $a[k] = 2h[(N/2)-k]$	**Type III Linear Phase Filter** $H(e^{j\omega}) = je^{-jN\omega/2} \sum_{k=1}^{N/2} c[k]\sin(k\omega)$ $c[k] = 2h[(N/2)-k]$
N odd	**Type II Linear Phase Filter** $H(e^{j\omega}) = e^{-jN\omega/2} \sum_{k=1}^{(N+1)/2} b[k]\cos((k-1/2)\omega)$ $b[k] = 2h[\dfrac{(N+1)}{2} - k]$	**Type IV Linear Phase Filter** $H(e^{j\omega}) = je^{-jN\omega/2} \sum_{k=1}^{(N+1)/2} d[k]\sin((k-1/2)\omega)$ $d[k] = 2h[\dfrac{(N+1)}{2} - k]$

Continued...

IIR Filter Design

Normalized Butterworth lowpass

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.000							
2	1.414	1.000						
3	2.000	2.000	1.000					
4	2.613	3.414	2.613	1.000				
5	3.236	5.236	5.236	3.236	1.000			
6	3.864	7.464	9.142	7.464	3.864	1.000		
7	4.494	10.10	14.59	14.59	10.10	4.494	1.000	
8	5.126	13.14	21.85	25.69	21.85	13.14	5.126	1.000

Filter order

$$d = \left(\frac{(1 - \delta_p)^{-2} - 1}{\delta_s^{-2} - 1} \right)^{0.5}$$

$$k = \frac{\Omega_p}{\Omega_s}$$

Design	Filter order
Butterworth	$N \geq \dfrac{\log d}{\log k}$
Chebyshev I, II	$N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$
Elliptic	$N \geq \dfrac{\log(16/d^2)}{\log(2/u)}$ $u = \dfrac{1 - (1 - k^2)^{0.25}}{1 + (1 - k^2)^{0.25}}$

$$\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$$

Continued...

Frequency transformations

Target class	Transformation	Edge frequencies of target class
Highpass	$s \rightarrow \dfrac{\Omega_p \Omega_p'}{s}$	Ω_p'
Bandpass	$s \rightarrow \Omega_p \dfrac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$	Ω_l, Ω_u
Bandstop	$s \rightarrow \Omega_p \dfrac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$	Ω_l, Ω_u

Impulse invariance transformation

$$H_a(s) = \sum_{k=0}^{p-1} \frac{A_k}{s - s_k} \longrightarrow H(z) = \sum_{k=0}^{p-1} \frac{T_s A_k}{1 - e^{s_k T_s} z^{-1}}$$

Bilinear transformation

$$H(z) = H_a(s) \Big|_{s = \frac{2}{T_s} \frac{1 - z^{-1}}{1 + z^{-1}}}$$

$$\Omega = 2 \tan(w/2)/T_s$$

Continued...

Discrete-time Fourier Analysis

The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1 x_1[n] + A_2 x_2[n]$	$A_1 X_1[k] + A_2 X_2[k]$
Time shifting	$x[\langle n - n_0 \rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0 n}$	$X[\langle k - k_0 \rangle_N]$
Time reversal	$x[\langle -n \rangle_N]$	$X[\langle -k \rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k \rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

End of Paper